**IMO 1975**

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**Day 1**

**1** We consider two sequences of real numbers $x_1 \geq x_2 \geq \ldots \geq x_n$ and $y_1 \geq y_2 \geq \ldots \geq y_n$. Let $z_1, z_2, \ldots, z_n$ be a permutation of the numbers $y_1, y_2, \ldots, y_n$. Prove that $\sum_{i=1}^{n}(x_i - y_i)^2 \leq \sum_{i=1}^{n}(x_i - z_i)^2$.

**2** Let $a_1, \ldots, a_n$ be an infinite sequence of strictly positive integers, so that $a_k < a_{k+1}$ for any $k$. Prove that there exists an infinity of terms $a_m$, which can be written like $a_m = x \cdot a_p + y \cdot a_q$ with $x, y$ strictly positive integers and $p \neq q$.

**3** In the plane of a triangle $ABC$, in its exterior, we draw the triangles $ABR, BCP, CAQ$ so that $\angle PBC = \angle CAQ = 45^\circ$, $\angle BCP = \angle QCA = 30^\circ$, $\angle ABR = \angle RAB = 15^\circ$.

Prove that

**a.)** $\angle QRP = 90\,^\circ$, and

**b.)** $QR = RP$.

**Day 2**

**4** When $4444^{4444}$ is written in decimal notation, the sum of its digits is $A$. Let $B$ be the sum of the digits of $A$. Find the sum of the digits of $B$. ($A$ and $B$ are written in decimal notation.)

**5** Can there be drawn on a circle of radius $1$ a number of $1975$ distinct points, so that the distance (measured on the chord) between any two points (from the considered points) is a rational number?

**6** Determine the polynomials P of two variables so that:

**a.)** for any real numbers $t, x, y$ we have $P(tx, ty) = t^n P(x, y)$ where $n$ is a positive integer, the same for all $t, x, y$;

**b.)** for any real numbers $a, b, c$ we have $P(a + b, c) + P(b + c, a) + P(c + a, b) = 0$;

**c.)** $P(1, 0) = 1$.