**IMO 1995**

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**Day 1** July 19th

**1** Let $A, B, C, D$ be four distinct points on a line, in that order. The circles with diameters $AC$ and $BD$ intersect at $X$ and $Y$. The line $XY$ meets $BC$ at $Z$. Let $P$ be a point on the line $XY$ other than $Z$. The line $CP$ intersects the circle with diameter $AC$ at $C$ and $M$, and the line $BP$ intersects the circle with diameter $BD$ at $B$ and $N$. Prove that the lines $AM, DN, XY$ are concurrent.

**2** Let $a$, $b$, $c$ be positive real numbers such that $abc = 1$. Prove that

$$\frac{1}{a^3\left(b+c\right)}+\frac{1}{b^3\left(c+a\right)}+\frac{1}{c^3\left(a+b\right)}\geq\frac{3}{2}.$$

**3** Determine all integers $n > 3$ for which there exist $n$ points $A_1, \cdots, A_n$ in the plane, no three collinear, and real numbers $r_1, \cdots, r_n$ such that for $1 \le i < j < k \le n$, the area of $\triangle A_iA_jA_k$ is $r_i + r_j + r_k$.

**Day 2** July 20th

**4** Find the maximum value of $x_0$ for which there exists a sequence $x_0, x_1 \cdots, x_{1995}$ of positive reals with $x_0 = x_{1995}$, such that

$$x_{i-1}+\frac{2}{x_{i-1}}=2x_i+\frac{1}{x_i},$$

for all $i = 1, \cdots, 1995$.

**5** Let $ABCDEF$ be a convex hexagon with $AB = BC = CD$ and $DE = EF = FA$, such that $\angle BCD = \angle EFA = \frac{\pi}{3}$. Suppose $G$ and $H$ are points in the interior of the hexagon such that $\angle AGB = \angle DHE = \frac{2\pi}{3}$. Prove that $AG + GB + GH + DH + HE \ge CF$.

**6** Let $p$ be an odd prime number. How many $p$-element subsets $A$ of $\{1, 2, \dots, 2p\}$ are there, the sum of whose elements is divisible by $p$?