Argentina Team Selection Test 2006

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Day 1

1 Let A be a set of natural numbers in which if a , b belong to A ($a > b$) then either $a + b$ or $a - b$ belong to A (both cases may be posible at the same time). Decide wheter there is or not a set A consisting on exactly 100 elements which has four elements x, y , z , w (not necesarilly distinct) that satisfy $x - y = 512$ and $z - w = 460$

Daniel

2 Given 365 cards, in which distinct numbers are written. We may ask for any three cards, the order of numbers written in them. Is it always possible to find out the order of all 365 cards by 2000 such questions?

3 In a circumference with center O we draw two equal chord $AB = CD$ and if $AB \cap CD = L$ then $AL > BL$ and $DL > CL$
We consider $M \in AL$ and $N \in DL$ such that $\widehat{ALC} = 2\widehat{MON}$
Prove that the chord determined by extending MN has the same as length as both AB and CD

Day 2

4 Find all integers solutions for $xy + yz + zx - xyz = 2$

5 Let p be a prime with $p > 5$, and let $S = \{p - n^2 | n \in \mathbb{N}, n^2 < p\}$. Prove that S contains two elements a and b such that $a|b$ and $1 < a < b$.

6 Let n be a natural number, and we consider the sequence $a_1, a_2 \ldots, a_{2n}$ where $a_i \in (-1, 0, 1)$
If we make the sum of consecutive members of the sequence, starting from one with an odd index and finishing in one with and even index, the result is ≤ 2 and ≥ -2
How many sequence are there satisfying this conditions?