

Argentina Team Selection Test 2008

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Day 1

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- 1** In the vertexes of a regular 100-gon we place the numbers from 1 to 100, in some order, every number appearing exactly once.
We say that an arrangement of the numbers is happy if for every symmetry axis of the polygon, the numbers which are from one side of the axis are greater than their respective symmetric (we don't take into consideration the numbers which are on the axis)
Find all happy arrangements (If two happy arrangements are the same under the rotation they are considered as only one)

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- 2** Triangle ABC is inscribed in a circumference Γ . A chord $MN = 1$ of Γ intersects the sides AB and AC at X and Y respectively, with M, X, Y, N in that order in MN . Let UV be the diameter of Γ perpendicular to MN with U and A in the same semiplane respect to MN . Lines AV , BU and CU cut MN in the ratios $\frac{3}{2}$, $\frac{4}{5}$ and $\frac{7}{6}$ respectively (starting counting from M). Find XY

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- 3** -Find all the functions from $\mathbb{R}^+ \rightarrow \mathbb{R}^+$ that satisfy: $x^2(f(x) + f(y)) = (x + y)(f(yf(x)))$

Day 2

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- 4** Let $d(n)$ be the number of positive divisors of the natural number n . Find all n such that $\frac{n}{d(n)} = p$ where p is a prime number

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- 5** Let ABC be a triangle, D , E and F the points of tangency of the incircle with sides BC , CA , AB respectively. Let P be the second point of intersection of CF and the incircle. If $ABPE$ is a cyclic quadrilateral prove that DP is parallel to AB

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- 6** Show that in acute triangle ABC we have: $\frac{a^5+b^5+c^5}{a^4+b^4+c^4} \geq \sqrt{3}R$
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