**National Math Olympiad (Second Round) 1986**

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**–** Analysis

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**1** Let $f$ be a function such that

$$f(x) = \frac{(x^2 - 2x + 1)\sin \frac{1}{x-1}}{\sin \pi x}.$$

Find the limit of $f$ in the point $x_0 = 1.$

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**2** **(a)** Sketch the diagram of the function $f$ if

$$f(x) = 4x(1 - |x|), \quad |x| \leq 1.$$

**(b)** Does there exist derivative of $f$ in the point $x = 0$ ?

**(c)** Let $g$ be a function such that

$$g(x) = \begin{cases} \frac{f(x)}{x} & : x \neq 0 \\ 4 & : x = 0 \end{cases}$$

Is the function $g$ continuous in the point $x = 0$ ?

**(d)** Sketch the diagram of $g.$

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**3** Find the smallest positive integer for which when we move the last right digit of the number to the left, the remaining number be $\frac{3}{2}$ times of the original number.

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**4** Find all positive integers $n$ for which the number $1! + 2! + 3! + \cdots + n!$ is a perfect power of an integer.

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**5** We have erasers, four pencils, two note books and three pens and we want to divide them between two persons so that every one receives at least one of the above stationery. In how many ways is this possible? [Note that the are not distinct.]

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**–** Geometry and Trigonometry

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**1** $O$ is a point in the plane. Let $O'$ be an arbitrary point on the axis $Ox$ of the plane and let $M$ be an arbitrary point. Rotate $M$, $90^\circ$ clockwise around $O$ to get the point $M'$ and rotate $M$, $90^\circ$ anticlockwise around $O'$ to get the point $M''$. Prove that the midpoint of the segment $MM''$ is a fixed point.

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**2** In a trapezoid $ABCD$, the legs $AB$ and $CD$ meet in $M$ and the diagonals $AC$ and $BD$ meet in $N$. Let $AC = a$ and $BC = b$. Find the area of triangles $AMD$ and $AND$ in terms of $a$ and $b$.

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**3** Prove that

$$\arctan \frac{1}{2} + \arctan \frac{1}{3} = \frac{\pi}{4}.$$

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