Spain Mathematical Olympiad 1964

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–   Day 1

1   Given the equation $x^2 + ax + 1 = 0$, determine:

a) The interval of possible values for a where the solutions to the previous equation are not real.
b) The loci of the roots of the polynomial, when a is in the previous interval.

2   The RTP tax is a function $f(x)$, where x is the total of the annual profits (in pesetas). Knowing that:

a) $f(x)$ is a continuous function
b) The derivative $\frac{df(x)}{dx}$ on the interval $0 \leq 6000$ is constant and equals zero; in the interval $6000 < x < P$ is constant and equals 1; and when $x > P$ is constant and equal 0.14.
c) $f(0) = 0$ and $f(140000) = 14000$.

Determine the value of the amount P (in pesetas) and represent graphically the function $y = f(x)$.

3   A convex polygon of n sides is considered. All its diagonals are drawn and we suppose that any three of them can only intersect on a vertex and that there is no pair of parallel diagonals. Under these conditions, we wish to compute

a) The total number of intersection points of these diagonals, excluding the vertices.

b) How many points, of these intersections, lie inside the polygon and how many lie outside.

4   We are given an equilateral triangle ABC, of side a, inside its circumscribed circle. We consider the smallest of the two portions of circle limited between AB and the circumference. If we consider parallel lines to BC, some of them cut the portion of circle in a segment. Which is the maximum possible length for one of the segments?

–   Day 2

5   Given a regular pentagon, its five diagonals are drawn. How many triangles do appear in the figure? Classify the set of triangles in classes of equal triangles.

6   Make a graphical representation of the function $y = |||x-1|-2|-3|$ on the interval $-8 \leq x \leq 8$.

7   A table with 1000 cards on a line, numbered from 1 to 1000, is considered. The cards are ordered in the usual way. Now, we proceed in the following way.

The first card (which is 1) is put just before the last card (between 999 and 1000) and, after, the new first card (which is 2) is put after the last card (which was 1000). Show that after 1000 movements, the cards are ordered again in the usual way. Show that the analogous result (n movements for n cards) does not hold when n is odd.

8   The points A and B lie on a horizontal line over a vertical plane. We consider the semicircumference passing through A and B that lies under the horizontal line. A segment of length a, with the same diameter that the semicircumference, moves in a way that always contains the point A and one of its extremes lies always on the semicircumference. Determine the value of the cosine of the angle between this segment and the horizontal line that makes the medium point of the segment to be as down as possible.