National Math Olympiad (Second Round) 1996

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1 Let a, b, c be real numbers. Prove that there exists a triangle with side lengths a, b, c if and only if

$$2(a^4 + b^4 + c^4) < (a^2 + b^2 + c^2)^2.$$

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2 Let a, b, c, d be positive integers such that $ab = cd$. Prove that $a + b + c + d$ is a composite number.

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3 Let N be the midpoint of side BC of triangle ABC. Right isosceles triangles ABM and ACP are constructed outside the triangle, with bases AB and AC. Prove that $\triangle MNP$ is also a right isosceles triangle.

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4 Let n blue points A_i and n red points B_i $(i = 1, 2, \ldots, n)$ be situated on a line. Prove that

$$\sum_{i,j} A_i B_j \geq \sum_{i<j} A_i A_j + \sum_{i<j} B_i B_j.$$
