National Math Olympiad (Second Round) 1997
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Day 1

1 Let x, y be positive integers such that $3x^2 + x = 4y^2 + y$. Prove that $x - y$ is a perfect square.

2 Let segments KN, KL be tangent to circle C at points N, L, respectively. M is a point on the extension of the segment KN and P is the other meet point of the circle C and the circumcircle of $\triangle KLM$. Q is on ML such that NQ is perpendicular to ML. Prove that

$$\angle MPQ = 2\angle KML.$$

3 We have a $n \times n$ table and weve written numbers $0, +1\ or\ -1$ in each 1×1 square such that in every row or column, there is only one $+1$ and one -1. Prove that by swapping the rows with each other and the columns with each other finitely, we can swap $+1$s with -1s.

Day 2

1 Let x_1, x_2, x_3, x_4 be positive reals such that $x_1x_2x_3x_4 = 1$. Prove that:

$$\sum_{i=1}^{4} x_i^3 \geq \max\{\sum_{i=1}^{4} x_i, \sum_{i=1}^{4} \frac{1}{x_i}\}.$$

2 In triangle ABC, angles B, C are acute. Point D is on the side BC such that $AD \perp BC$. Let the interior bisectors of $\angle B, \angle C$ meet AD at E, F, respectively. If $BE = CF$, prove that ABC is isosceles.

3 Let a, b be positive integers and $p = \frac{b}{4}\sqrt{\frac{2a-b}{2a+b}}$ be a prime number. Find the maximum value of p and justify your answer.