National Math Olympiad (Second Round) 2003
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Day 1

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1 We call the positive integer n a $3-$*stratum* number if we can divide the set of its positive divisors into 3 subsets such that the sum of each subset is equal to the others. $a)$ Find a $3-$stratum number. $b)$ Prove that there are infinitely many $3-$stratum numbers.

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2 In a village, there are n houses with $n>2$ and all of them are not collinear. We want to generate a water resource in the village. For doing this, point A is *better* than point B if the sum of the distances from point A to the houses is less than the sum of the distances from point B to the houses. We call a point *ideal* if there doesnt exist any *better* point than it. Prove that there exist at most 1 *ideal* point to generate the resource.

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3 n volleyball teams have competed to each other (each 2 teams have competed exactly 1 time.). For every 2 distinct teams like A,B, there exist exactly t teams which have lost their match with A,B. Prove that $n=4t+3$. (Notabene that in volleyball, there doesnt exist tie!)

Day 2

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1 Let $x,y,z\in\mathbb{R}$ and $xyz=-1$. Prove that:

$$x^4+y^4+z^4+3(x+y+z)\geq\frac{x^2}{y}+\frac{x^2}{z}+\frac{y^2}{x}+\frac{y^2}{z}+\frac{z^2}{x}+\frac{z^2}{y}.$$

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2 $\angle A$ is the least angle in ΔABC. Point D is on the arc BC from the circumcircle of ΔABC. The perpendicular bisectors of the segments AB,AC intersect the line AD at M,N, respectively. Point T is the meet point of BM,CN. Suppose that R is the radius of the circumcircle of ΔABC. Prove that:

$$BT+CT\leq 2R.$$

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3 We have a chessboard and we call a 1×1 square a room. A robot is standing on one arbitrary vertex of the rooms. The robot starts to move and in every one movement, he moves one side of a room. This robot has 2 memories A,B. At first, the values of A,B are 0. In each movement, if he goes up, 1 unit is added to A, and if he goes down, 1 unit is waned from A, and if he goes right, the value of A is added to B, and if he goes left, the value of A is waned from B. Suppose that the robot has traversed a traverse (!) which hasnt intersected itself and finally, he has come back to its initial vertex. If $v(B)$ is the value of B in the last of the traverse, prove that in this traverse, the interior surface of the shape that the robot has moved on its circumference is equal to $|v(B)|$.
