National Math Olympiad (Second Round) 2007

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Day 1

1 In triangle ABC, $\angle A = 90^\circ$ and M is the midpoint of BC. Point D is chosen on segment AC such that $AM = AD$ and P is the second meet point of the circumcircles of triangles $\Delta AMC, \Delta BDC$. Prove that the line CP bisects $\angle ACB$.

2 Two vertices of a cube are A, O such that AO is the diagonal of one its faces. A $n-$run is a sequence of $n+1$ vertices of the cube such that each 2 consecutive vertices in the sequence are 2 ends of one side of the cube. Is the $1386-$runs from O to itself less than $1386-$runs from O to A or more than it?

3 In a city, there are some buildings. We say the building A is dominant to the building B if the line that connects upside of A to upside of B makes an angle more than 45° with earth. We want to make a building in a given location. Suppose none of the buildings are dominant to each other. Prove that we can make the building with a height such that again, none of the buildings are dominant to each other. (Suppose the city as a horizontal plain and each building as a perpendicular line to the plain.)

Day 2

1 Prove that for every positive integer n, there exist n positive integers such that the sum of them is a perfect square and the product of them is a perfect cube.

2 Tow circles C, D are exterior tangent to each other at point P. Point A is in the circle C. We draw 2 tangents AM, AN from A to the circle D (M, N are the tangency points.). The second meet points of AM, AN with C are E, F, respectively. Prove that $\frac{PE}{PF} = \frac{ME}{NF}$.

3 Farhad has made a machine. When the machine starts, it prints some special numbers. The property of this machine is that for every positive integer n, it prints exactly one of the numbers $n, 2n, 3n$. We know that the machine prints 2. Prove that it doesn't print 13824.