National Math Olympiad (Second Round) 2010

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1 Let a,b be two positive integers and $a>b$.We know that $\gcd(a-b,ab+1)=1$ and $\gcd(a+b,ab-1)=1$. Prove that $(a-b)^2+(ab+1)^2$ is not a perfect square.

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2 There are n points in the page such that no three of them are collinear.Prove that number of triangles that vertices of them are chosen from these n points and area of them is 1,is not greater than $\frac{2}{3}(n^2-n)$.

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3 Circles W_1,W_2 meet at $D$and P. A and B are on W_1,W_2 respectively, such that AB is tangent to W_1 and W_2. Suppose D is closer than P to the line AB. AD meet circle W_2 for second time at C. Let M be the midpoint of BC. Prove that $\angle DPM=\angle BDC$.

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4 Let $P(x)=ax^3+bx^2+cx+d$ be a polynomial with real coefficients such that

$$\min\{d,b+d\}>\max\{|c|,|a+c|\}$$

Prove that $P(x)$ do not have a real root in $[-1,1]$.

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5 In triangle ABC we havev $\angle A=\frac{\pi}{3}$. Construct E and F on continue of AB and AC respectively such that $BE=CF=BC$. Suppose that EF meets circumcircle of $\triangle ACE$ in K. ($K\not\equiv E$). Prove that K is on the bisector of $\angle A$.

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6 A school has n students and some super classes are provided for them. Each student can participate in any number of classes that he/she wants. Every class has at least two students participating in it. We know that if two different classes have at least two common students, then the number of the students in the first of these two classes is different from the number of the students in the second one. Prove that the number of classes is not greater that $(n-1)^2$.
