National Math Olympiad (Second Round) 2011

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by goodar2006

	Day 1
1	We have a line and 1390 points around it such that the distance of each point to the line is less than 1 centimeters and the distance between any two points is more than 2 centimeters. prove that there are two points such that their distance is at least 10 meters (1000 centimeters).
2	In triangle ABC, we have $\angle ABC = 60$. The line through B perpendicular to side AB intersects angle bisector of $\angle BAC$ in D and the line through C perpendicular BC intersects angle bisector of $\angle ABC$ in E. prove that $\angle BED \leq 30$.
3	Find all increasing sequences $a_1, a_2, a_3, ...$ of natural numbers such that for each $i, j \in \mathbb{N}$, number of the divisors of $i+j$ and $a_i + a_j$ is equal. (an increasing sequence is a sequence that if $i \leq j$, then $a_i \leq a_j$.)
	Day 2
1	find the smallest natural number n such that there exists n real numbers in the interval $(-1, 1)$ such that their sum equals zero and the sum of their squares equals 20.
2	rainbow is the name of a bird. this bird has n colors and it's colors in two consecutive days are not equal. there doesn't exist 4 days in this bird's life like i, j, k, l such that $i < j < k < l$ and the bird has the same color in days i and k and the same color in days j and l different from the colors it has in days i and k. what is the maximum number of days rainbow can live in terms of n?
3	The line l intersects the extension of AB in D (D is nearer to B than A) and the extension of AC in E (E is nearer to C than A) of triangle ABC. Suppose that reflection of line l to perpendicular bisector of side BC intersects the mentioned extensions in D' and E' respectively. Prove that if $BD + CE = DE$, then $BD' + CE' = D'E'$.