www.artofproblemsolving.com/community/c389682
by l1090107005, dcouchman, sqing, termas

– Let p and $p+2$ be primes, $p>3$. Sequence $\{a_n\}: a_1=2, a_n=a_{n-1}+\lceil \frac{pa_{n-1}}{n} \rceil$. Prove that $n \mid pa_{n-1}+1$ for all $n=3,4,\ldots,p-1$.

– Test 1

Q10 Let $f(x)$ is an odd function on R, $f(1)=1$ and $f(\frac{x}{x-1})=xf(x)$ $(\forall x<0)$.
Find the value of $f(1)f(\frac{1}{100})+f(\frac{1}{2})f(\frac{1}{99})+f(\frac{1}{3})f(\frac{1}{98})+\cdots+f(\frac{1}{50})f(\frac{1}{51})$.

– Test 2

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1 Let $a_1,a_2,\ldots,a_{2016}$ be real numbers such that $9a_i \ge 11a_{i+1}^2$ $(i=,2,\cdots,2015)$.
Find the maximum value of $(a_1-a_2^2)(a_2-a_3^2)\cdots(a_{2015}-a_{2016}^2)(a_{2016}-a_1^2)$.

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2 Let X,Y be two points which lies on the line BC of $\triangle ABC(X,B,C,Y$ lies in sequence$)$ such that $BX\cdot AC=CY\cdot AB$, O_1,O_2 are the circumcenters of $\triangle ACX, \triangle ABY$, $O_1O_2\cap AB=U, O_1O_2\cap AC=V$. Prove that $\triangle AUV$ is a isosceles triangle.

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3 Given 10 points in the space such that each 4 points are not lie on a plane. Connect some points with some segments such that there are no triangles or quadrangles. Find the maximum number of the segments.

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4 Let $p>3$ and $p+2$ are prime numbers,and define sequence

$$a_1=2, a_n=a_{n-1}+\lfloor \frac{pa_{n-1}}{n} \rfloor$$

show that:for any $n=3,4,\cdots,p-1$ have

$$n|pa_{n-1}+1$$

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