BMO TST 2010

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by ridgers

1 **a)** Is the number $1111\cdots 11$ (with 2010 ones) a prime number?
b) Prove that every prime factor of $1111\cdots 11$ (with 2011 ones) is of the form $4022j+1$ where j is a natural number.

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2 Let $a\geq 2$ be a real number; with the roots x_1 and x_2 of the equation $x^2-ax+1=0$ we build the sequence with $S_n=x_1^n+x_2^n$.
a)Prove that the sequence $\frac{S_n}{S_{n+1}}$, where n takes value from 1 up to infinity, is strictly non increasing.
b)Find all value of a for the which this inequality hold for all natural values of n $\frac{S_1}{S_2}+\cdots+\frac{S_n}{S_{n+1}}>n-1$

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3 Let K be the circumscribed circle of the trapezoid $ABCD$. In this trapezoid the diagonals AC and BD are perpendicular. The parallel sides $AB=a$ and $CD=c$ are diameters of the circles K_a and K_b respectively. Find the perimeter and the area of the part inside the circle K, that is outside circles K_a and K_b.

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4 Let's consider the inequality $a^3+b^3+c^3<k(a+b+c)(ab+bc+ca)$ where a,b,c are the sides of a triangle and k a real number.
a) Prove the inequality for $k=1$.
b) Find the smallest value of k such that the inequality holds for all triangles.
