BMO TST 2011

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by ridgers

1 The given parabola $y = ax^2 + bx + c$ doesn't intersect the X-axis and passes from the points $A(-2, 1)$ and $B(2, 9)$. Find all the possible values of the x coordinates of the vertex of this parabola.

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2 The area and the perimeter of the triangle with sides $10, 8, 6$ are equal. Find all the triangles with integral sides whose area and perimeter are equal.

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3 In the acute angle triangle ABC the point O is the center of the circumscribed circle and the lines OA, OB, OC intersect sides BC, CA, AB respectively in points M, N, P such that $\angle NMP = 90^o$.

(a) Find the ratios $\frac{\angle AMN}{\angle NMC}, \frac{\angle AMP}{\angle PMB}$.

(b) If any of the angles of the triangle ABC is 60^o, find the two other angles.

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4 Find all prime numbers p such that $2^p + p^2$ is also a prime number.

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5 The sweeties shop called "Olympiad" sells boxes of $6, 9$ or 20 chocolates. Groups of students from a school that is near the shop collect money to buy a chocolate for each student; to make this they buy a box and than give to everybody a chocolate. Like this students can create groups of $15 = 6 + 9$ students, $38 = 2 * 9 + 20$ students, etc. The seller has promised to the students that he can satisfy any group of students, and if he will need to open a new box of chocolate for any group (like groups of $4, 7$ or 10 students) than he will give all the chocolates for free to this group. Can there be constructed the biggest group that profits free chocolates, and if so, how many students are there in this group?
