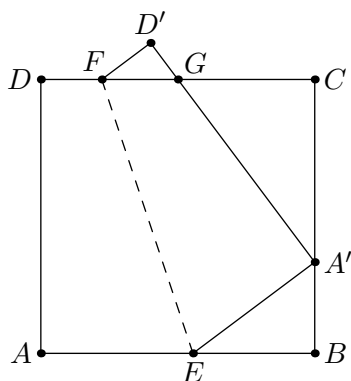


India National Olympiad 2017
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- 1 In the given figure, $ABCD$ is a square sheet of paper. It is folded along EF such that A goes to a point A' different from B and C , on the side BC and D goes to D' . The line $A'D'$ cuts CD in G . Show that the inradius of the triangle GCA' is the sum of the inradii of the triangles $GD'F$ and $A'BE$.



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- 2 Suppose $n \geq 0$ is an integer and all the roots of $x^3 + \alpha x + 4 - (2 \times 2016^n) = 0$ are integers. Find all possible values of α .
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- 3 Find the number of triples (x, a, b) where x is a real number and a, b belong to the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ such that
- $$x^2 - a\{x\} + b = 0.$$
- where $\{x\}$ denotes the fractional part of the real number x .
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- 4 Let $ABCDE$ be a convex pentagon in which $\angle A = \angle B = \angle C = \angle D = 120^\circ$ and the side lengths are five consecutive integers in some order. Find all possible values of $AB + BC + CD$.
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- 5 Let ABC be a triangle with $\angle A = 90^\circ$ and $AB < AC$. Let AD be the altitude from A on to BC . Let P, Q and I denote respectively the incentres of triangle ABD, ACD and ABC . Prove that AI is perpendicular to PQ and $AI = PQ$.
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- 6 Let $n \geq 1$ be an integer and consider the sum

$$x = \sum_{k \geq 0} \binom{n}{2k} 2^{n-2k} 3^k = \binom{n}{0} 2^n + \binom{n}{2} 2^{n-2} \cdot 3 + \binom{n}{4} 2^{n-4} \cdot 3^2 + \dots$$

Show that $2x-1$, $2x$, $2x+1$ form the sides of a triangle whose area and inradius are also integers.
