

Kosovo National Mathematical Olympiad 2016

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– Grade 9

1 If $a, b \neq 0$ are real numbers such that $a^2b^2(a^2b^2 + 4) = 2(a^6 + b^6)$, then show that a, b cant be both of them rational .

2 Find all real numbers x which satisfied $|2x + 1| + |x - 1| = 2 - x$.

3 Show that the sum $S = 5 + 5^2 + 5^3 + \dots + 5^{2016}$ is divisible by 31

4 In a planet *Papella* year has 400 days with days counting from 1 – 400 . A holiday would be that day which is divisible by 6 . The new gonerment decide to reform a new calendar and split in 10 months with 40 day each month , and they decide that day of month which is divisible by 6 to be holiday . Show that after reform the number of holidays after one year decreased less then 10 percent .

5 It is given rectangle $ABCD$ with length $|AB| = 15cm$ and with length of altitude $|BE| = 12cm$ where BC is altitude of triangle ABC . Find perimeter and area of rectangle $ABCD$.

– Grade 10

1 Find all triples (x, y, z) of integers such that satisfied:

$$x^2 + y^2 + z^2 + xy + yz + zx = 6$$

2 Show that the number $2017^{2016} - 2016^{2017}$ is divisible by 5 .

3 The distance from A to B is $408km$. From A in direction of B move motorcyclist , and from B in direction of A move a bicyclist . If a motorcyclist start to move 2 hours earlier then bycyclist , then they will meet 7 hours after bicyclist start to move . If a bicyclist start to move 2 hours earlier then motorcyclist , then they will meet 8 hours after after motorcyclist start to move . Find the velocity of motorcyclist and bicyclist if we now that the velocity of them was constant all the time .

4 Let be $f : (0, +\infty) \rightarrow \mathbb{R}$ monoton-decreasing .
If $f(2a^2 + a + 1) < f(3a^2 - 4a + 1)$ find interval of a .

5 If a, b, c are sides of right triangle with c hypotenuse then show that for every positive integer $n > 2$ we have $c^n > a^n + b^n$.

– Grade 11

1 Find all couples (m, n) of positive integers such that satisfied $m^2 + 1 = n^2 + 2016$.

2 Evaluate the sum of all three digits number which are not divisible by 13.

3 If α is an acute angle and $a, b \geq 0$ then show that:

$$\left(a + \frac{b}{\sin \alpha}\right) \left(b + \frac{a}{\cos \alpha}\right) \geq a^2 + b^2 + 3ab$$

4 Solve equation in real numbers $\log_2(4^x + 4) = x + \log_2(2^{x+1} - 3)$

5 In angle $\angle AOB = 60^\circ$ are two circle which circumscribed and tangjent to each other. If we write with r and R the radius of smaller and bigger circle respectively and if $r = 1$ find R .

– Grade 12

1 Find all three digit numbers such that the square of that number is equal to the sum of their digits in power of 5.

2 Sum of all coefficients of polynomial $P(x)$ is equal with 2. Also the sum of coefficients which are at odd exponential in x^k are equal to sum of coefficients which are at even exponential in x^k . Find the residue of polynomial $P(x)$ when it is divide by $x^2 - 1$.

3 Let be a, b, c complex numbers such that $|a| = |b| = |c| = r$ then show that

$$\left| \frac{ab+bc+ca}{a+b+c} \right| = r$$

4 In all rectangles with same diagonal d find that one with bigger area.

5 In trapezoid $ABCD$ with AB parallel to CD show that :

$$\frac{|AB|^2 - |BC|^2 + |AC|^2}{|CD|^2 - |AD|^2 + |AC|^2} = \frac{|AB|}{|CD|} = \frac{|AB|^2 - |AD|^2 + |BD|^2}{|CD|^2 - |BC|^2 + |BD|^2}$$