Albania Team Selection Test 2010

www.artofproblemsolving.com/community/c3966

by ridgers

1 ABC is an acute angle triangle such that $AB > AC$ and $B\hat{A}C = 60^{\circ}$. Let's denote by O the center of the circumscribed circle of the triangle and H the intersection of altitudes of this triangle. Line OH intersects AB in point P and AC in point Q. Find the value of the ration $\frac{PO}{HQ}$.

---

2 Find all the continuous functions $f : \mathbb{R} \mapsto \mathbb{R}$ such that $\forall x, y \in \mathbb{R}$, $(1 + f(x)f(y))f(x + y) = f(x) + f(y)$.

---

3 One point of the plane is called $rational$ if both coordinates are rational and $irrational$ if both coordinates are irrational. Check whether the following statements are true or false:
a) Every point of the plane is in a line that can be defined by 2 rational points.
b) Every point of the plane is in a line that can be defined by 2 irrational points.

This maybe is not algebra so sorry if I putted it in the wrong category!

---

4 With $\sigma(n)$ we denote the sum of natural divisors of the natural number n. Prove that, if n is the product of different prime numbers of the form $2^k - 1$ for $k \in \mathbb{N}$($Mersenne's$ prime numbers) , than $\sigma(n) = 2^m$, for some $m \in \mathbb{N}$. Is the inverse statement true?

---

5 **a)** Let's consider a finite number of big circles of a sphere that do not pass all from a point. Show that there exists such a point that is found only in two of the circles. (With big circle we understand the circles with radius equal to the radius of the sphere.)
b) Using the result of part $a)$ show that, for a set of n points in a plane, that are not all in a line, there exists a line that passes through only two points of the given set.
