Albania Team Selection Test 2013

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1   Find the 3-digit number whose ratio with the sum of its digits it's minimal.

2   Let a, b, c, d be positive real numbers such that $abcd = 1$. Find with proof that $x = 3$ is the minimal value for which the following inequality holds:

$$a^x + b^x + c^x + d^x \geq \frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}$$

3   Solve the function $f : \Re \to \Re$:

$$f(x^3) + f(y^3) = (x + y)(f(x^2) + f(y^2) - f(xy))$$

4   It is given a triangle ABC whose circumcenter is O and orthocenter H.
        If $AO = AH$ find the angle $B\hat{A}C$ of that triangle.

5   Let k be a natural number. Find all the couples of natural numbers (n, m) such that : $(2^k)! = 2^n * m$