Albania National Olympiad 2010

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by ridgers

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1   Let A and B be two fixed points of a given circle and XY a diameter of this circle. Find the locus of the intersection points of lines AX and BY . (BY is not a diameter of the circle).

Albanian National Mathematical Olympiad 2010−12 GRADE Question 1.

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2   We denote $N_{2010} = \{1, 2, \cdots, 2010\}$
(a) How many non empty subsets does this set have?
(b) For every non empty subset of the set N_{2010} we take the product of the elements of the subset. What is the sum of these products?
(c) Same question as the **(b)** part for the set $-N_{2010} = \{-1, -2, \cdots, -2010\}$.

Albanian National Mathematical Olympiad 2010−12 GRADE Question 2.

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3   **(a)**Prove that every pentagon with integral coordinates has at least two vertices , whose respective coordinates have the same parity.
(b) What is the smallest area possible of pentagons with integral coordinates.

Albanian National Mathematical Olympiad 2010−12 GRADE Question 3.

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4   The sequence of Fibonnaci's numbers if defined from the two first digits $f_1 = f_2 = 1$ and the formula $f_{n+2} = f_{n+1} + f_n, \forall n \in N$.
(a) Prove that f_{2010} is divisible by 10.
(b) Is f_{1005} divisible by 4?

Albanian National Mathematical Olympiad 2010−12 GRADE Question 4.

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5   All members of the senate were firstly divided into S senate commissions . According to the rules, no commission has less that 5 senators and every two commissions have different number of senators. After the first session the commissions were closed and new commissions were opened. Some of the senators now are not a part of any commission. It resulted also that every two senators that were in the same commission in the first session , are not any more in the same commission.
(a) Prove that at least $4S + 10$ senators were left outside the commissions.
(b) Prove that this number is achievable.

Albanian National Mathematical Olympiad 2010−12 GRADE Question 5.