

Albania National Olympiad 2011

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by ridgers

- 1 (a) Find the minimal distance between the points of the graph of the function $y = \ln x$ from the line $y = x$.
(b) Find the minimal distance between two points, one of the point is in the graph of the function $y = e^x$ and the other point in the graph of the function $y = \ln x$.

 - 2 Find all the values that can take the last digit of a "perfect" even number. (The natural number n is called "perfect" if the sum of all its natural divisors is equal twice the number itself. For example: the number 6 is perfect ,because $1 + 2 + 3 + 6 = 2 \cdot 6$).

 - 3 In a convex quadrilateral $ABCD$, $\angle ABC$ and $\angle BCD$ are $\geq 120^\circ$. Prove that $|AC| + |BD| \geq |AB| + |BC| + |CD|$. (With $|XY|$ we understand the length of the segment XY).

 - 4 The sequence (a_n) is defined by $a_1 = 1$ and $a_n = n(a_1 + a_2 + \dots + a_{n-1})$, $\forall n > 1$.
(a) Prove that for every even n , a_n is divisible by $n!$.
(b) Find all odd numbers n for the which a_n is divisible by $n!$.

 - 5 The triangle ABC acute with gravity center M is such that $\angle AMB = 2\angle ACB$. Prove that:
(a) $AB^4 = AC^4 + BC^4 - AC^2 \cdot BC^2$,
(b) $\angle ACB \geq 60^\circ$.
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