Albania National Olympiad 2012

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by mhet49

1   Find all primes p such that $p + 2$ and $p^2 + 2p - 8$ are also primes.

2   The trinomial $f(x)$ is such that $(f(x))^3 - f(x) = 0$ has three real roots. Find the y-coordinate of the vertex of $f(x)$.

3   Let S_i be the sum of the first i terms of the arithmetic sequence $a_1, a_2, a_3 \ldots$. Show that the value of the expression

$$\frac{S_i}{i}(j - k) + \frac{S_j}{j}(k - i) + \frac{S_k}{k}(i - j)$$

does not depend on the numbers i, j, k nor on the choice of the arithmetic sequence $a_1, a_2, a_3, \ldots$.

4   Find all functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$f(x^3) + f(y^3) = (x + y)f(x^2) + f(y^2) - f(xy)$$

for all $x \in \mathbb{R}$.

5   Let ABC be a triangle where $AC \neq BC$. Let P be the foot of the altitude taken from C to AB; and let V be the orthocentre, O the circumcentre of ABC, and D the point of intersection between the radius OC and the side AB. The midpoint of CD is E.

a) Prove that the reflection V' of V in AB is on the circumcircle of the triangle ABC.

b) In what ratio does the segment EP divide the segment OV?