

Benelux 2011

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- 1** An ordered pair of integers (m, n) with $1 < m < n$ is said to be a *Benelux couple* if the following two conditions hold: m has the same prime divisors as n , and $m+1$ has the same prime divisors as $n+1$.
- (a) Find three Benelux couples (m, n) with $m \leq 14$.
- (b) Prove that there are infinitely many Benelux couples
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- 2** Let ABC be a triangle with incentre I . The angle bisectors AI , BI and CI meet $[BC]$, $[CA]$ and $[AB]$ at D , E and F , respectively. The perpendicular bisector of $[AD]$ intersects the lines BI and CI at M and N , respectively. Show that A, I, M and N lie on a circle.
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- 3** If k is an integer, let $c(k)$ denote the largest cube that is less than or equal to k . Find all positive integers p for which the following sequence is bounded: $a_0 = p$ and $a_{n+1} = 3a_n - 2c(a_n)$ for $n \geq 0$.
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- 4** Abby and Brian play the following game: They first choose a positive integer N . Then they write numbers on a blackboard in turn. Abby starts by writing a 1. Thereafter, when one of them has written the number n , the other writes down either $n+1$ or $2n$, provided that the number is not greater than N . The player who writes N on the blackboard wins.
- (a) Determine which player has a winning strategy if $N = 2011$.
- (b) Find the number of positive integers $N \leq 2011$ for which Brian has a winning strategy.
- (This is based on ISL 2004, Problem C5.)
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