Benelux 2014

www.artofproblemsolving.com/community/c3991

by TheMaskedMagician

–	May 3rd
1	Find the smallest possible value of the expression $$\left\lfloor\frac{a+b+c}{d}\right\rfloor+\left\lfloor\frac{b+c+d}{a}\right\rfloor+\left\lfloor\frac{c+d+a}{b}\right\rfloor+\left\lfloor\frac{d+a+b}{c}\right\rfloor$$ in which $a,\ b,\ c$, and d vary over the set of positive integers. (Here $\lfloor x\rfloor$ denotes the biggest integer which is smaller than or equal to x.)
2	Let $k\geq 1$ be a positive integer. We consider $4k$ chips, $2k$ of which are red and $2k$ of which are blue. A sequence of those $4k$ chips can be transformed into another sequence by a so-called move, consisting of interchanging a number (possibly one) of consecutive red chips with an equal number of consecutive blue chips. For example, we can move from $r\underline{bb}br\underline{rr}b$ to $r\underline{rr}br\underline{bb}b$ where r denotes a red chip and b denotes a blue chip. Determine the smallest number n (as a function of k) such that starting from any initial sequence of the $4k$ chips, we need at most n moves to reach the state in which the first $2k$ chips are red.
3	For all integers $n\geq 2$ with the following property: - for each pair of positive divisors $k,\ \ell<n$, at least one of the numbers $2k-\ell$ and $2\ell-k$ is a (not necessarily positive) divisor of n as well.
4	Let $ABCD$ be a square. Consider a variable point P inside the square for which $\angle BAP\geq 60^\circ$. Let Q be the intersection of the line AD and the perpendicular to BP in P. Let R be the intersection of the line BQ and the perpendicular to BP from C. - **(a)** Prove that $\lvert BP\rvert\geq\lvert BR\rvert$ - **(b)** For which point(s) P does the inequality in **(a)** become an equality?

© 2019 AoPS Incorporated