Spain Mathematical Olympiad 1998

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Day 1

1 A unit square $ABCD$ with centre O is rotated about O by an angle α. Compute the common area of the two squares.

2 Find all four-digit numbers which are equal to the cube of the sum of their digits.

3 Let ABC be a triangle. Points D and E are taken on the line BC such that AD and AE are parallel to the respective tangents to the circumcircle at C and B. Prove that

$$\frac{BE}{CD} = \left(\frac{AB}{AC}\right)^2$$

Day 2

1 Find the tangents of the angles of a triangle knowing that they are positive integers.

2 Find all strictly increasing functions $f : \mathbb{N} \to \mathbb{N}$ that satisfy

$$f(n + f(n)) = 2f(n) \quad \text{for all } n \in \mathbb{N}$$

3 Determine the values of n for which an $n \times n$ square can be tiled with pieces of the type `http://oi53.tinypic.com/v3pqoh.jpg`.