

**Spain Mathematical Olympiad 2008**

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by WakeUp

**Day 1**

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1 Find two positive integers  $a$  and  $b$ , when their sum and their least common multiple is given. Find the numbers when the sum is 3972 and the least common multiple is 985928.

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2 Let  $a$  and  $b$  be two real numbers, with  $0 < a, b < 1$ . Prove that

$$\sqrt{ab^2 + a^2b} + \sqrt{(1-a)(1-b)^2 + (1-a)^2(1-b)} < \sqrt{2}$$

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3 Let  $p \geq 3$  be a prime number. Each side of a triangle is divided into  $p$  equal parts, and we draw a line from each division point to the opposite vertex. Find the maximum number of regions, every two of them disjoint, that are formed inside the triangle.

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**Day 2**

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1 Let  $p$  and  $q$  be two different prime numbers. Prove that there are two positive integers,  $a$  and  $b$ , such that the arithmetic mean of the divisors of  $n = p^a q^b$  is an integer.

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2 Given a circle, two fixed points  $A$  and  $B$  and a variable point  $P$ , all of them on the circle, and a line  $r$ ,  $PA$  and  $PB$  intersect  $r$  at  $C$  and  $D$ , respectively. Find two fixed points on  $r$ ,  $M$  and  $N$ , such that  $CM \cdot DN$  is constant for all  $P$ .

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3 Every point in the plane is coloured one of seven distinct colours. Is there an inscribed trapezoid whose vertices are all of the same colour?

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