Spain Mathematical Olympiad 2009

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Day 1

1 Find all the finite sequences with n consecutive natural numbers $a_1, a_2, ..., a_n$, with $n \geq 3$ such that $a_1 + a_2 + ... + a_n = 2009$.

2 Let ABC be an acute triangle with the incircle $C(I, r)$ and the circumcircle $C(O, R)$. Denote $D \in BC$ for which $AD \perp BC$ and $AD = h_a$. Prove that $DI^2 = (2R - h_a)(h_a - 2r)$.

3 Some edges are painted in red. We say that a coloring of this kind is *good*, if for each vertex of the polyhedron, there exists an edge which concurs in that vertex and is not painted red. Moreover, we say that a coloring where some of the edges of a regular polyhedron is *completely good*, if in addition to being *good*, no face of the polyhedron has all its edges painted red. What regular polyhedrons is equal the maximum number of edges that can be painted in a *good* color and a *completely good*? Explain your answer.

Day 2

4 Find all the integer pairs (x, y) such that:

$$x^2 - y^4 = 2009$$

5 Let, a, b, c real positive numbers with $abc = 1$
Prove:

$(\frac{a}{1+ab})^2 + (\frac{b}{1+bc})^2 + (\frac{c}{1+ca})^2 \geq \frac{3}{4}$

Thanks!

6 Inside a circle of center O and radius r, take two points A and B symmetrical about O. We consider a variable point P on the circle and draw the chord $\overline{PP'} \perp \overline{AP}$. Let C is the symmetric of B about $\overline{PP'}$ ($\overline{PP}'$ is the axis of symmetry) . Find the locus of point $Q = \overline{PP'} \cap \overline{AC}$ when we change P in the circle.