The second Pakistan Team Selection Test

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Problem 1 Let $ABCD$ be a cyclic quadrilateral. The diagonals AC and BD meet at P, and DA and CB meet at Q. Suppose PQ is perpendicular to AC. Let E be the midpoint of AB. Prove that PE is perpendicular to BC.

Problem 2 There are n students in a circle, one behind the other, all facing clockwise. The students have heights $h_1 < h_2 < h_3 < \cdots < h_n$. If a student with height h_k is standing directly behind a student with height h_{k-2} or lesss, the two students are permitted to switch places Prove that it is not possible to make more than $\binom{n}{3}$ such switches before reaching a position in which no further switches are possible.

Problem 3 Find all $f : \mathbb{R}^+ \to \mathbb{R}^+$ such that for all distinct x, y, z

$$f(x)^2 - f(y)f(z) = f(x^y)f(y)f(z)[f(y^z) - f(z^x)]$$
