

NMO 2008

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Day 1

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- 1 What is the maximum number of triangles with vertices on the points of the fork/graph which is possible to construct?
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- 2 Let $AEBC$ be a cyclic quadrilateral. Let D be a point on the ray AE which is outside the circumscribed circumference of $AEBC$. Suppose that $\angle CAB = \angle BAE$. Prove that $AB = BD$ if and only if $DE = AC$.
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- 3 Let d be a natural number. Given two natural numbers M and N with d digits, M is a friend of N if and only if the d numbers obtained substituting each one of the digits of M by the digit of N which is on the same position are all multiples of 7. Find all the values of d for which the following condition is valid:
For any two numbers M and N with d digits, M is a friend of N if and only if N is a friend of M .
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Day 2

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- 4 Nelson challenges Telma for the following game:
First Telma takes 2^9 numbers from the set $\{0, 1, 2, 3, \dots, 1024\}$, then Nelson takes 2^8 of the remaining numbers. Then Telma takes 2^7 numbers and successively, until only two numbers remain. Nelson will have to give Telma the difference between these two numbers in euros. What is the largest amount Telma can win, whatever Nelson's strategy is?
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- 5 Let ABC be a right-angled triangle in A such that $AB < AC$. Let M be the midpoint of BC and let D be the intersection of AC with the perpendicular line to BC which passes through M . Let E be the intersection point of the parallel line to AC which passes through M with the perpendicular line to BD which passes through B . Prove that triangles AEM and MCA are similar if and only if $\angle ABC = 60^\circ$.
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- 6 Let n be a natural number larger than 2. Vanessa has n piles of jade stones, and all the piles have a different number of stones. Vanessa can distribute the stones from any pile by the other piles and stay with $n - 1$ piles with the same number of stones. She also can distribute the stones from any two piles by the other piles and stay with $n - 2$ piles with the same number of stones. Find the smallest possible number of jade's stones that the pile with the largest number of stones can have.
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