

**NMO 2009**

[www.artofproblemsolving.com/community/c4036](http://www.artofproblemsolving.com/community/c4036)

by Joao Pedro Santos

**Day 1**

- 
- 1 A circumference was divided in  $n$  equal parts. On each of these parts one number from 1 to  $n$  was placed such that the distance between consecutive numbers is always the same. Numbers 11, 4 and 17 were in consecutive positions. In how many parts was the circumference divided?
- 
- 2 Points  $N$  and  $M$  are on the sides  $CD$  and  $BC$  of square  $ABCD$ , respectively. The perimeter of triangle  $MCN$  is equal to the double of the length of the square's side. Find  $\angle MAN$ .
- 
- 3 Duarte wants to draw a square whose side's length is 2009 cm and which is divided in  $2009 \times 2009$  squares whose side's length is 1 cm and whose sides are parallel to the original square's one, without taking the pencil out of the paper. Starting on one of the vertex of the giant square, what is the length of the shortest line that allows him to make this drawing?
- 

**Day 2**

- 
- 1 Joo calculated the product of the non zero digits of each integer from 1 to  $10^{2009}$  and then he summed these  $10^{2009}$  products. Which number did he obtain?
- 
- 2 Circumferences  $C_1$  and  $C_2$  have different radii and are externally tangent on point  $T$ . Consider points  $A$  on  $C_1$  and  $B$  on  $C_2$ , both different from  $T$ , such that  $\angle BTA = 90^\circ$ . What is the locus of the midpoints of line segments  $AB$  constructed that way?
- 
- 3 Two players play the following game on a circular board with 2009 houses. The two players put, alternatively, on an empty house, one of three pieces, called *explorer* ( $E$ ), *trap* ( $T$ ) or *stone* ( $S$ ). A treasure is a sequence of three consecutive filled houses such that the first one (on any direction) has an explorer and the middle one doesn't have a trap. For example,  $STE$  is not a treasure, while  $TEE$  is a treasure. The first player forming a treasure wins. Can any of the players guarantee the victory? And, in affirmative case, who?
-