

NMO 2010

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by Joao Pedro Santos

Day 1

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- 1 There are several candles of the same size on the Chapel of Bones. On the first day a candle is lit for a hour. On the second day two candles are lit for a hour, on the third day three candles are lit for a hour, and successively, until the last day, when all the candles are lit for a hour. On the end of that day, all the candles were completely consumed. Find all the possibilities for the number of candles.
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- 2 On a circumference, points A and B are on opposite arcs of diameter CD . Line segments CE and DF are perpendicular to AB such that $A - E - F - B$ (i.e., A, E, F and B are collinear on this order). Knowing $AE = 1$, find the length of BF .
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- 3 On each day, more than half of the inhabitants of vora eats *sericaia* as dessert. Show that there is a group of 10 inhabitants of vora such that, on each of the last 2010 days, at least one of the inhabitants ate *sericaia* as dessert.
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Day 2

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- 1 Giraldo wrote five distinct natural numbers on the vertices of a pentagon. And next he wrote on each side of the pentagon the least common multiple of the numbers written of the two vertices who were on that side and noticed that the five numbers written on the sides were equal. What is the smallest number Giraldo could have written on the sides?
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- 2 Show that any triangle has two sides whose lengths a and b satisfy $\frac{\sqrt{5}-1}{2} < \frac{a}{b} < \frac{\sqrt{5}+1}{2}$.
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- 3 Consider a square $(p-1) \times (p-1)$, where p is a prime number, which is divided by squares 1×1 whose sides are parallel to the initial square's sides. Show that it is possible to select p vertices such that there are no three collinear vertices.
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