Kazakhstan National Olympiad 2009

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Day 1

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1 Let S_n be number of ordered sets of natural numbers $(a_1; a_2; ....; a_n)$ for which $\frac{1}{a_1}+\frac{1}{a_2}+....+\frac{1}{a_n}=1$. Determine
1)$S_{10} mod(2)$.
2)$S_7 mod(2)$.

(1) is first problem in 10 grade, (2)- third in 9 grade.

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2 Let in-circle of ABC touch AB, BC, AC in C_1, A_1, B_1 respectively.
Let H- intersection point of altitudes in $A_1B_1C_1$, I and O-be in-center and circumcenter of ABC respectively.
Prove, that I, O, H lies on one line.

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3 In chess tournament participates n participants ($n > 1$). In tournament each of participants plays with each other exactly 1 game. For each game participant have 1 point if he wins game, $0,5$ point if game is drow and 0 points if he lose game.
If after ending of tournament participant have at least 75 of maximum possible points he called $winner\ of\ tournament$.
Find maximum possible numbers of $winners\ of\ tournament$.

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4 Let a, b, c, d-reals positive numbers. Prove inequality: $\frac{a^2+b^2+c^2}{ab+bc+cd}+\frac{b^2+c^2+d^2}{bc+cd+ad}+\frac{a^2+c^2+d^2}{ab+ad+cd}+\frac{a^2+b^2+d^2}{ab+ad+bc}\geq 4$

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5 Quadrilateral $ABCD$ inscribed in circle with center O. Let lines AD and BC intersects at M, lines AB and CD- at N, lines AC and BD -at P, lines OP and MN at K.
Proved that $\angle AKP = \angle PKC$.

As I know, this problem was very short solution by polars, but in olympiad for this solution gives maximum 4 balls (in marking schemes written, that needs to prove all theorems about properties of polars)

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6 Let $P(x)$ be polynomial with integer coefficients.
Prove, that if for any natural k holds equality: $\underbrace{P(P(...P(0)...))}_{n-times}=0$ then $P(0)=0$ or $P(P(0))=0$

Day 2

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1 Prove that for any natural $n \geq 2$, the number $\underbrace{2^{2^{\dots^2}}}_{n \text{ times}} - \underbrace{2^{2^{\dots^2}}}_{n-1 \text{ times}}$ is divisible by n.

I know, that it is a very old problem :blush: but it is a problem from olympiad.

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2 In triangle ABC $AA_1; BB_1; CC_1$-altitudes. Let I_1 and I_2 be in-centers of triangles AC_1B_1 and CA_1B_1 respectively. Let in-circle of ABC touch AC in B_2.
Prove, that quadrilateral $I_1I_2B_1B_2$ inscribed in a circle.

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Find maximum possible numbers of $winners\ of\ tournament$.

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4 Let $0 < a_1 \leq a_2 \leq \dots \leq a_n$ ($n \geq 3; n \in \mathbb{N}$) be n real numbers. Prove the inequality

$$\frac{a_1^2}{a_2} + \frac{a_2^3}{a_3^2} + \dots + \frac{a_n^{n+1}}{a_1^n} \geq a_1 + a_2 + \dots + a_n$$

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6 Is there exist four points on plane, such that distance between any two of them is integer odd number?

May be it is geometry or number theory or combinatoric, I don't know, so it here :blush:

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