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- 1 In a given triangle ABC , O is its circumcenter, D is the midpoint of AB and E is the centroid of the triangle ACD . Show that the lines CD and OE are perpendicular if and only if $AB = AC$.

- 2 Let $a, b, c, d \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ be real numbers such that $\sin a + \sin b + \sin c + \sin d = 1$ and $\cos 2a + \cos 2b + \cos 2c + \cos 2d \geq \frac{10}{3}$.
Prove that $a, b, c, d \in [0, \frac{\pi}{6}]$.

- 3 Let S be the set of all positive integers of the form $19a + 85b$, where a, b are arbitrary positive integers. On the real axis, the points of S are colored in red and the remaining integer numbers are colored in green. Find, with proof, whether or not there exists a point A on the real axis such that any two points with integer coordinates which are symmetrical with respect to A have necessarily distinct colors.

- 4 There are 1985 participants to an international meeting. In any group of three participants there are at least two who speak the same language. It is known that each participant speaks at most five languages. Prove that there exist at least 200 participants who speak the same language.
