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- 1 Let a be a real number and let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function satisfying $f(0) = \frac{1}{2}$ and

$$f(x+y) = f(x)f(a-y) + f(y)f(a-x), \quad \forall x, y \in \mathbb{R}.$$

Prove that f is constant.

- 2 Find all real numbers x, y greater than 1, satisfying the condition that the numbers $\sqrt{x-1} + \sqrt{y-1}$ and $\sqrt{x+1} + \sqrt{y+1}$ are nonconsecutive integers.
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- 3 In the triangle ABC the following equality holds:

$$\sin^{23} \frac{A}{2} \cos^{48} \frac{B}{2} = \sin^{23} \frac{B}{2} \cos^{48} \frac{A}{2}$$

Determine the value of $\frac{AC}{BC}$.

- 4 Two circles K_1 and K_2 , centered at O_1 and O_2 with radii 1 and $\sqrt{2}$ respectively, intersect at A and B . Let C be a point on K_2 such that the midpoint of AC lies on K_1 . Find the length of the segment AC if $O_1O_2 = 2$
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