Balkan MO 1988

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1 Let ABC be a triangle and let M, N, P be points on the line BC such that AM, AN, AP are the altitude, the angle bisector and the median of the triangle, respectively. It is known that $\frac{[AMP]}{[ABC]} = \frac{1}{4}$ and $\frac{[ANP]}{[ABC]} = 1 - \frac{\sqrt{3}}{2}$.
Find the angles of triangle ABC.

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2 Find all polynomials of two variables $P(x, y)$ which satisfy

$$P(a, b)P(c, d) = P(ac + bd, ad + bc), \forall a, b, c, d \in \mathbb{R}.$$

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3 Let $ABCD$ be a tetrahedron and let d be the sum of squares of its edges' lengths. Prove that the tetrahedron can be included in a region bounded by two parallel planes, the distances between the planes being at most $\frac{\sqrt{d}}{2\sqrt{3}}$

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4 Let $(a_n)_{n\geq 1}$ be a sequence defined by $a_n = 2^n + 49$. Find all values of n such that $a_n = pg, a_{n+1} = rs$, where p, q, r, s are prime numbers with $p < q, r < s$ and $q - p = s - r$.
