Balkan MO 1992

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by Valentin Vornicu

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1	For all positive integers m,n define $f(m,n)=m^{3^{4n}+6}-m^{3^{4n}+4}-m^5+m^3$. Find all numbers n with the property that $f(m,n)$ is divisible by 1992 for every m. *Bulgaria*
2	Prove that for all positive integers n the following inequality takes place $$(2n^2+3n+1)^n\geq 6^n(n!)^2.$$ *Cyprus*
3	Let D, E, F be points on the sides BC, CA, AB respectively of a triangle ABC (distinct from the vertices). If the quadrilateral $AFDE$ is cyclic, prove that $$\frac{4\mathcal{A}[DEF]}{\mathcal{A}[ABC]}\leq\left(\frac{EF}{AD}\right)^2.$$ *Greece*
4	For each integer $n\geq 3$, find the least natural number $f(n)$ having the property $\star$ For every $A\subset\{1,2,\ldots,n\}$ with $f(n)$ elements, there exist elements $x,y,z\in A$ that are pairwise coprime.