

Balkan MO 2000

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- 1 Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ such that

$$f(xf(x) + f(y)) = f^2(x) + y$$

for all $x, y \in \mathbb{R}$.

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- 2 Let ABC be an acute-angled triangle and D the midpoint of BC . Let E be a point on segment AD and M its projection on BC . If N and P are the projections of M on AB and AC then the interior angle bisectors of $\angle NMP$ and $\angle NEP$ are parallel.
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- 3 How many $1 \times 10\sqrt{2}$ rectangles can be cut from a 50×90 rectangle using cuts parallel to its edges?
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- 4 Show that for any n we can find a set X of n distinct integers greater than 1, such that the average of the elements of any subset of X is a square, cube or higher power.
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