Balkan MO 2003

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1	Can one find 4004 positive integers such that the sum of any 2003 of them is not divisible by 2003?
2	Let ABC be a triangle, and let the tangent to the circumcircle of the triangle ABC at A meet the line BC at D. The perpendicular to BC at B meets the perpendicular bisector of AB at E. The perpendicular to BC at C meets the perpendicular bisector of AC at F. Prove that the points D, E and F are collinear. *Valentin Vornicu*
3	Find all functions $f : \mathbb{Q} \to \mathbb{R}$ which fulfill the following conditions: a) $f(1) + 1 > 0$; b) $f(x+y) - xf(y) - yf(x) = f(x)f(y) - x - y + xy$, for all $x, y \in \mathbb{Q}$; c) $f(x) = 2f(x+1) + x + 2$, for every $x \in \mathbb{Q}$.
4	A rectangle $ABCD$ has side lengths $AB = m$, $AD = n$, with m and n relatively prime and both odd. It is divided into unit squares and the diagonal AC intersects the sides of the unit squares at the points $A_1 = A, A_2, A_3, \ldots, A_k = C$. Show that $$A_1A_2 - A_2A_3 + A_3A_4 - \cdots + A_{k-1}A_k = \frac{\sqrt{m^2+n^2}}{mn}.$$