

Balkan MO 2006

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1 Let a, b, c be positive real numbers. Prove the inequality

$$\frac{1}{a(b+1)} + \frac{1}{b(c+1)} + \frac{1}{c(a+1)} \geq \frac{3}{1+abc}.$$

2 Let ABC be a triangle and m a line which intersects the sides AB and AC at interior points D and F , respectively, and intersects the line BC at a point E such that C lies between B and E . The parallel lines from the points A, B, C to the line m intersect the circumcircle of triangle ABC at the points A_1, B_1 and C_1 , respectively (apart from A, B, C). Prove that the lines A_1E , B_1F and C_1D pass through the same point.

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3 Find all triplets of positive rational numbers (m, n, p) such that the numbers $m + \frac{1}{np}$, $n + \frac{1}{pm}$, $p + \frac{1}{mn}$ are integers.

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4 Let m be a positive integer and $\{a_n\}_{n \geq 0}$ be a sequence given by $a_0 = a \in \mathbb{N}$, and

$$a_{n+1} = \begin{cases} \frac{a_n}{2} & \text{if } a_n \equiv 0 \pmod{2}, \\ a_n + m & \text{otherwise.} \end{cases}$$

Find all values of a such that the sequence is periodical (starting from the beginning).
