Ukraine National Mathematical Olympiad 2014
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by hajimbrak

–	Grade level 8
–	Grade level 9
–	Grade level 10
Day 1	
1	Suppose that for real x, y, z, t the following equalities hold:$\{x+y+z\} = \{y+z+t\} = \{z+t+x\} = \{t+x+y\} = 1/4$. Find all possible values of $\{x+y+z+t\}$.(Here$\{x\} = x - [x]$)
2	Let M be the midpoint of the side BC of $\triangle ABC$. On the side AB and AC the points E and F are chosen. Let K be the point of the intersection of BF and CE and L be chosen in a way that $CL \parallel AB$ and $BL \parallel CE$. Let N be the point of intersection of AM and CL. Show that KN is parallel to FL. **Edit**:Fixed typographical error.
3	It is known that for natural numbers a, b, c, d and n the following inequalities hold: $a + c < n$ and $a/b + c/d < 1$. Prove that $a/b + c/d < 1 - 1/n^3$.
4	There are 100 cards with numbers from 1 to 100 on the table.Andriy and Nick took the same number of cards in a way such that the following condition holds:if Andriy has a card with a number n then Nick has a card with a number $2n + 2$.What is the maximal number of cards that could be taken by the two guys?
Day 2	
1	Find the values of x such that the following inequality holds: $\min\{\sin x, \cos x\} < \min\{1 - \sin x, 1 - \cos x\}$
2	Find all pairs of prime numbers p and q that satisfy the equation $3p^q - 2q^{p-1} = 19$.
3	Is it possible to choose 24 points in the space,such that no three of them lie on the same line and choose 2013 planes in such a way that each plane passes through at least 3 of the chosen points and each triple of points belongs to at least one of the chosen planes?

4 Let M be the midpoint of the internal bisector AD of $\triangle ABC$.Circle ω_1 with diameter AC intersects BM at E and circle ω_2 with diameter AB intersects CM at F.Show that B, E, F, C are concyclic.

– Grade level 11