Kyrgyzstan National Olympiad 2009

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by theSA

1 a, b, c are sides of triangle ABC. For any choosen triple from $(a+1, b, c), (a, b+1, c), (a, b, c+1)$ there exist a triangle which sides are choosen triple. Find all possible values of area which triangle ABC can take.

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2 x and y are real numbers. $A)$ If it is known that $x+y$ and $x+y^2$ are rational numbers, can we conclude that x and y are also rational numbers. $B)$ If it is known that $x+y$, $x+y^2$ and $x+y^3$ are rational numbers, can we conclude that x and y are also rational numbers.

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3 For function $f: \mathbb{R} \to \mathbb{R}$ given that $f(x^2+x+3)+2\cdot f(x^2-3x+5)=6x^2-10x+17$, calculate $f(2009)$.

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4 Find all real (x, y) such that
$x+y^2=y^3$ $y+x^2=x^3$

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5 Prove for all natural n that $40^n \cdot n! | (5n)!$

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6 Find all natural a, b such that $a(a+b)+1 | (a+b)(b+1)-1$.

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7 Does $a^2+b^2+c^2 \leqslant 2(ab+bc+ca)$ hold for every a, b, c if it is known that $a^4+b^4+c^4 \leqslant 2(a^2b^2+b^2c^2+c^2a^2)$.

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8 Does there exist a function $f: \mathbb{N} \to \mathbb{N}$ such that $f(f(n-1)) = f(n+1)-f(n)$ for all $n > 2$.

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9 For any positive $a_1, a_2, ..., a_n$ prove that $\frac{a_1}{a_2+a_3}+\frac{a_2}{a_3+a_4}+...+\frac{a_n}{a_1+a_2} > \frac{n}{4}$ holds.
