Kyrgyzstan National Olympiad 2011

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by theSA

1 For a given chord MN of a circle discussed the triangle ABC, whose base is the diameter AB of this circle,which do not intersect the MN, and the sides AC and BC pass through the ends of M and N of the chord MN. Prove that the heights of all such triangles ABC drawn from the vertex C to the side AB, intersect at one point.

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2 In a convex n-gon all angles are equal from a certain point, located inside the n-gon, all its sides are seen under equal angles. Can we conclude that this n-gon is regular?

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3 Given positive numbers $a_1, a_2, ..., a_n$ with $a_1+a_2+...+a_n = 1$. Prove that $\left(\frac{1}{a_1^2}-1\right)\left(\frac{1}{a_2^2}-1\right)...\left(\frac{1}{a_n^2}-1\right) \geqslant (n^2-1)^n$.

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4 Given equation $a^5 - a^3 + a = 2$, with real a . Prove that $3 < a^6 < 4$.

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5 Points M and N are chosen on sides AB and BC,respectively, in a triangle ABC, such that point O is interserction of lines CM and AN. Given that $AM + AN = CM + CN$. Prove that $AO + AB = CO + CB$.

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6 **a)** Among the 21 pairwise distances between the 7 points of the plane, prove that one and the same number occurs not more than 12 times.

b) Find a maximum number of times may meet the same number among the 15 pairwise distances between 6 points of the plane.

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7 Given that $g(n) = \frac{1}{2+\frac{1}{3+\frac{1}{...+\frac{1}{n-1}}}}$ and $k(n) = \frac{1}{2+\frac{1}{3+\frac{1}{...+\frac{1}{n-1+\frac{1}{n}}}}}$, for natural n. Prove that $|g(n) - k(n)| \leq \frac{1}{(n-1)!n!}$.

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8 Given a sequence $x_1, x_2, ..., x_n$ of real numbers with ${x_{n+1}}^3 = {x_n}^3 - 3{x_n}^2 + 3x_n$, where $(n = 1, 2, 3, ...)$. What must be value of x_1, so that x_{100} and x_{1000} becomes equal?
