

Kosovo Team Selection Test 2011www.artofproblemsolving.com/community/c4102

by Obel1x

- 1 Let a, b, c be real positive numbers. Prove that the following inequality holds:

$$\sum_{\text{cyc}} \sqrt{\frac{5a^2 + 5c^2 + 8b^2}{4ac}} \geq 3 \cdot \sqrt[9]{\frac{8(a+b)^2(b+c)^2(c+a)^2}{(abc)^2}}$$

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- 2 Prove that the lines joining the middle-points of non-adjacent sides of an convex quadrilateral and the line joining the middle-points of diagonals, are concurrent. Prove that the intersection point is the middle point of the three given segments.

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- 3 Let n be a natural number, for which we define $S(n) = \{1 + g + g^2 + \dots + g^{n-1} | g \in \mathbb{N}, g \geq 2\}$

a) Prove that: $S(3) \cap S(4) = \emptyset$

b) Determine: $S(3) \cap S(5)$

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- 4 From the number 7^{1996} we delete its first digit, and then add the same digit to the remaining number. This process continues until the left number has ten digits. Show that the left number has two same digits.

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- 5 Find all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ such that $\forall x \notin \{-1, 1\}$ holds:

$$f\left(\frac{x-3}{x+1}\right) + f\left(\frac{3+x}{1-x}\right) = x$$