

**Kosovo Team Selection Test 2012**

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by Arber1

1 A student had 18 papers. He selected some of these papers, then he cut each of them in 18 pieces. He took these pieces and selected some of them, which he again cut in 18 pieces each. The student took this procedure until he got tired. After a time he counted the pieces and got 2012 pieces. Prove that the student was wrong during the counting.

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2 Find all three digit numbers, for which the sum of squares of each digit is 90.

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3 If  $a, b, c$  are the sides of a triangle and  $m_a, m_b, m_c$  are the medians prove that

$$4(m_a^2 + m_b^2 + m_c^2) = 3(a^2 + b^2 + c^2)$$

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4 Each term in a sequence 1, 0, 1, 0, 1, 0... starting with the seventh is the sum of the last 6 terms mod 10. Prove that the sequence ..., 0, 1, 0, 1, 0, 1... never occurs

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5 Prove that the equation

$$\frac{4}{n} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

has infinitely many natural solutions

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