

**Morocco National Olympiad 2005**

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**1** In a square  $ABCD$  let  $F$  be the midpoint of  $[CD]$  and let  $E$  be a point on  $[AB]$  such that  $AE > EB$ . The parallel with  $(DE)$  passing by  $F$  meets the segment  $[BC]$  at  $H$ . Prove that the line  $(EH)$  is tangent to the circle circumscribed with  $ABCD$ .

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**2** Find all the positive integers  $x, y, z$  satisfying:  $x^2 + y^2 + z^2 = 2xyz$

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**3** Consider  $n$  points  $A_1, A_2, \dots, A_n$  on a circle. How many ways are there if we want to color these points by  $p$  colors, so that each two neighbors points are colored with two different colors?

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**4** 21 distinct numbers are chosen from the set  $\{1, 2, 3, \dots, 2046\}$ . Prove that we can choose three distinct numbers  $a, b, c$  among those 21 numbers such that

$$bc < 2a^2 < 4bc$$