APMO 1992

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1 A triangle with sides a, b, and c is given. Denote by s the semiperimeter, that is $s = \frac{a+b+c}{2}$. Construct a triangle with sides $s-a$, $s-b$, and $s-c$. This process is repeated until a triangle can no longer be constructed with the side lengths given.

For which original triangles can this process be repeated indefinitely?

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2 In a circle C with centre O and radius r, let C_1, C_2 be two circles with centres O_1, O_2 and radii r_1, r_2 respectively, so that each circle C_i is internally tangent to C at A_i and so that C_1, C_2 are externally tangent to each other at A.

Prove that the three lines OA, O_1A_2, and O_2A_1 are concurrent.

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3 Let n be an integer such that $n > 3$. Suppose that we choose three numbers from the set $\{1, 2, \ldots, n\}$. Using each of these three numbers only once and using addition, multiplication, and parenthesis, let us form all
possible combinations.
(a) Show that if we choose all three numbers greater than $\frac{n}{2}$, then the values of these combinations are all distinct.
(b) Let p be a prime number such that $p \leq \sqrt{n}$. Show that the number of ways of choosing three numbers so that the smallest one is p and the values of the combinations are not all distinct is precisely the number of positive divisors of $p-1$.

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4 Determine all pairs (h, s) of positive integers with the following property:

If one draws h horizontal lines and another s lines which satisfy
(i) they are not horizontal,
(ii) no two of them are parallel,
(iii) no three of the $h+s$ lines are concurrent,

then the number of regions formed by these $h+s$ lines is 1992.

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5 Find a sequence of maximal length consisting of non-zero integers in which the sum of any seven consecutive terms is positive and that of any eleven consecutive terms is negative.
