APMO 1995

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1 Determine all sequences of real numbers a_1, a_2, ..., a_{1995} which satisfy:

$$2\sqrt{a_n-(n-1)} \geq a_{n+1}-(n-1), \text{ for } n=1,2,\dots 1994,$$

and

$$2\sqrt{a_{1995}-1994} \geq a_1+1.$$

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2 Let a_1, a_2, ..., a_n be a sequence of integers with values between 2 and 1995 such that:
(i) Any two of the a_i's are relatively prime,
(ii) Each a_i is either a prime or a product of primes.
Determine the smallest possible values of n to make sure that the sequence will contain a prime number.

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3 Let $PQRS$ be a cyclic quadrilateral such that the segments PQ and RS are not parallel. Consider the set of circles through P and Q, and the set of circles through R and S. Determine the set A of points of tangency of circles in these two sets.

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4 Let C be a circle with radius R and centre O, and S a fixed point in the interior of C. Let AA' and BB' be perpendicular chords through S. Consider the rectangles $SAMB$, $SBN'A'$, $SA'M'B'$, and $SB'NA$. Find the set of all points M, N', M', and N when A moves around the whole circle.

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5 Find the minimum positive integer k such that there exists a function f from the set $\mathbb{Z}$ of all integers to $\{1,2,\dots k\}$ with the property that $f(x) \neq f(y)$ whenever $|x-y| \in \{5,7,12\}$.

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