**APMO 2011**

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by WakeUp

**1** Let $a, b, c$ be positive integers. Prove that it is impossible to have all of the three numbers $a^2 + b + c, b^2 + c + a, c^2 + a + b$ to be perfect squares.

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**2** Five points $A_1, A_2, A_3, A_4, A_5$ lie on a plane in such a way that no three among them lie on a same straight line. Determine the maximum possible value that the minimum value for the angles $\angle A_iA_jA_k$ can take where $i, j, k$ are distinct integers between $1$ and $5$.

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**3** Let $ABC$ be an acute triangle with $\angle BAC = 30^\circ$. The internal and external angle bisectors of $\angle ABC$ meet the line $AC$ at $B_1$ and $B_2$, respectively, and the internal and external angle bisectors of $\angle ACB$ meet the line $AB$ at $C_1$ and $C_2$, respectively. Suppose that the circles with diameters $B_1B_2$ and $C_1C_2$ meet inside the triangle $ABC$ at point $P$. Prove that $\angle BPC = 90^\circ$ .

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**4** Let $n$ be a fixed positive odd integer. Take $m + 2$ **distinct** points $P_0, P_1, \ldots, P_{m+1}$ (where $m$ is a non-negative integer) on the coordinate plane in such a way that the following three conditions are satisfied:
1) $P_0 = (0, 1), P_{m+1} = (n + 1, n)$, and for each integer $i, 1 \leq i \leq m$, both $x$- and $y$- coordinates of $P_i$ are integers lying in between $1$ and $n$ ($1$ and $n$ inclusive).
2) For each integer $i, 0 \leq i \leq m$, $P_iP_{i+1}$ is parallel to the $x$-axis if $i$ is even, and is parallel to the $y$-axis if $i$ is odd.
3) For each pair $i, j$ with $0 \leq i < j \leq m$, line segments $P_iP_{i+1}$ and $P_jP_{j+1}$ share at most $1$ point.
Determine the maximum possible value that $m$ can take.

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**5** Determine all functions $f : \mathbb{R} \to \mathbb{R}$, where $\mathbb{R}$ is the set of all real numbers, satisfying the following two conditions:
1) There exists a real number $M$ such that for every real number $x, f(x) < M$ is satisfied.
2) For every pair of real numbers $x$ and $y$,

$$f(xf(y)) + yf(x) = xf(y) + f(xy)$$

is satisfied.

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