Malaysia National Olympiad 2010

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– Bongsu

1 A square with side length 2 cm is placed next to a square with side length 6 cm, as shown in the diagram. Find the shaded area, in cm^2.
`https://cdn.artofproblemsolving.com/attachments/5/7/ceb4912a6e73ca751113b2b5c92cbfdbb6e0c`
`png`

2 A student wrote down the following sequence of numbers : the first number is 1, the second number is 2, and after that, each number is obtained by adding together all the previous numbers. Determine the 12th number in the sequence.

3 Adam has RM2010 in his bank account. He donates RM10 to charity every day. His first donation is on Monday. On what day will he donate his last RM10?

4 In the diagram, $\angle AOB = \angle BOC$ and$\angle COD = \angle DOE = \angle EOF$. Given that $\angle AOD = 82^o$ and $\angle BOE = 68^o$. Find $\angle AOF$.
`https://cdn.artofproblemsolving.com/attachments/b/2/deba6cd740adbf033ad884fff8e13cd21d9c5`
`png`

5 A circle and a square overlap such that the overlapping area is 50% of the area of the circle, and is 25% of the area of the square, as shown in the figure. Find the ratio of the area of the square outside the circle to the area of the whole figure.
`https://cdn.artofproblemsolving.com/attachments/e/2/c209a95f457dbf3c46f66f82c0a45cc4b5c1c`
`png`

6 Find the number of different pairs of positive integers (a, b) for which $a + b \leq 100$ and

$$\frac{a + \frac{1}{b}}{\frac{1}{a} + b} = 10$$

7 Let ABC be a triangle in which $AB = AC$. A point I lies inside the triangle such that $\angle ABI = \angle CBI$ and $\angle BAI = \angle CAI$. Prove that

$$\angle BIA = 90^o + \frac{\angle C}{2}$$

8 Find the last digit of

$$7^1 \times 7^2 \times 7^3 \times \cdots \times 7^{2009} \times 7^{2010}.$$

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9 A number of runners competed in a race. When Ammar finished, there were half as many runners who had finished before him compared to the number who finished behind him. Julia was the 10th runner to finish behind Ammar. There were twice as many runners who had finished before Julia compared to the number who finished behind her. How many runners were there in the race?

– Muda

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1 Triangles OAB, OBC, OCD are isoceles triangles with $\angle OAB = \angle OBC = \angle OCD = \angle 90^o$. Find the area of the triangle OAB if the area of the triangle OCD is 12.

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2 A meeting is held at a round table. It is known that 7 women have a woman on their right side, and 12 women have a man on their right side. It is also known that 75

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3 Let $\gamma = \alpha \times \beta$ where

$$\alpha = 999 \cdots 9$$

(2010 '9') and

$$\beta = 444 \cdots 4$$

(2010 '4')
Find the sum of digits of γ.

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4 A square $ABCD$ has side length 1. A circle passes through the vertices of the square. Let P, Q, R, S be the midpoints of the arcs which are symmetrical to the arcs AB, BC, CD, DA when reflected on sides AB, BC, CD, DA, respectively. The area of square $PQRS$ is $a + b\sqrt{2}$, where a and b are integers. Find the value of $a + b$.
`https://cdn.artofproblemsolving.com/attachments/4/3/fc9e1bd71b26cfd9ff076db7aa0a396ae64e7`
`png`

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5 Find the number of triples of nonnegative integers (x, y, z) such that

$$x^2 + 2xy + y^2 - z^2 = 9.$$

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6 A two-digit integer is divided by the sum of its digits. Find the largest remainder that can occur.

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7 Let ABC be a triangle in which $AB = AC$ and let I be its incenter. It is known that $BC = AB + AI$. Let D be a point on line BA extended beyond A such that $AD = AI$. Prove that $DAIC$ is a cyclic quadrilateral.

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8 For any number x, let $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x. A sequence $a_1, a_2, \cdots$ is given, where

$$a_n = \left\lfloor \sqrt{2n} + \frac{1}{2} \right\rfloor.$$

How many values of k are there such that $a_k = 2010$?

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9 Let m and n be positive integers such that $2^n + 3^m$ is divisible by 5. Prove that $2^m + 3^n$ is divisible by 5.

– Sulung

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1 In the diagram, congruent rectangles $ABCD$ and $DEFG$ have a common vertex D. Sides BC and EF meet at H. Given that $DA = DE = 8$, $AB = EF = 12$, and $BH = 7$. Find the area of $ABHED$.
`https://cdn.artofproblemsolving.com/attachments/f/b/7225fa89097e7b20ea246b3aa920d2464080a png`

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2 Find x such that

$$2010^{\log_{10} x} = 11^{\log_{10}(1+3+5+\cdots+4019)}.$$

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3 Let $N = \overline{abc}$ be a three-digit number. It is known that we can construct an isoceles triangle with a, b and c as the length of sides. Determine how many possible three-digit number N there are.

($N = \overline{abc}$ means that a, b and c are digits of N, and not $N = a \times b \times c$.)

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4 A semicircle has diameter XY. A square $PQRS$ with side length 12 is inscribed in the semicircle with P and S on the diameter. Square $STUV$ has T on RS, U on the semicircle, and V on XY. What is the area of $STUV$?

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5 Let n be an integer greater than 1. If all digits of $97n$ are odd, find the smallest possible value of n.

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6 Find the smallest integer $k \geq 3$ with the property that it is possible to choose two of the number $1, 2, ..., k$ in such a way that their product is equal to the sum of the remaining $k - 2$ numbers.

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7 A line segment of length 1 is given on the plane. Show that a line segment of length $\sqrt{2010}$ can be constructed using only a straightedge and a compass.

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8 Show that

$$\log_a bc + \log_b ca + \log_c ab \geq 4(\log_{ab} c + \log_{bc} a + \log_{ca} b)$$

for all a, b, c greater than 1.

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9 Show that there exist integers m and n such that

$$\frac{m}{n} = \sqrt[3]{\sqrt{50}+7} - \sqrt[3]{\sqrt{50}-7}.$$