Rioplatense Mathematical Olympiad, Level 3 2005

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Day 1

1 Find all numbers n that can be expressed in the form $n=k+2\lfloor\sqrt{k}\rfloor+2$ for some nonnegative integer k.

2 In trapezoid $ABCD$, the sum of the lengths of the bases AB and CD is equal to the length of the diagonal BD. Let M denote the midpoint of BC, and let E denote the reflection of C about the line DM. Prove that $\angle AEB=\angle ACD$.

3 Find the largest positive integer n not divisible by 10 which is a multiple of each of the numbers obtained by deleting two consecutive digits (neither of them in the first or last position) of n. (Note: n is written in the usual base ten notation.)

Day 2

1 Let P be a point inside triangle ABC and let R denote the circumradius of triangle ABC. Prove that

$$\frac{PA}{AB\cdot AC}+\frac{PB}{BC\cdot BA}+\frac{PC}{CA\cdot CB}\ge\frac{1}{R}.$$

2 Consider all finite sequences of positive real numbers each of whose terms is at most 3 and the sum of whose terms is more than 100. For each such sequence, let S denote the sum of the subsequence whose sum is the closest to 100, and define the *defect* of this sequence to be the value $|S-100|$. Find the maximum possible value of the defect.

3 Let k be a positive integer. Show that for all $n>k$ there exist convex figures $F_1,\dots,F_n$ and F such that there doesn't exist a subset of k elements from $F_1,...,F_n$ and F is covered for this elements, but F is covered for every subset of $k+1$ elements from $F_1,F_2,.....,F_n$.